Preface

Fractals and Point Processes

Fractals are objects that possess a form of self-scaling; a part of the whole can be made to recreate the whole by shifting and stretching. Many objects are self-scaling only in a statistical sense, meaning that a part of the whole can be made to recreate the whole in the likeness of their probability distributions, rather than as exact replicas. Examples of random fractals include the length of a segment of coastline, the variation of water flow in the river Nile, and the human heart rate.

Point processes are mathematical representations of random phenomena whose individual events are largely identical and occur principally at discrete times and locations. Examples include the arrival of cars at a tollbooth, the release of neurotransmitter molecules at a biological synapse, and the sequence of human heartbeats.

Fractals began to find their way into the scientific literature some 50 years ago. For point processes this took place perhaps 100 years ago, although both concepts developed far earlier. These two fields of study have grown side-by-side, reflecting their increasing importance in the natural and technological worlds. However, the domains in which point processes and fractals both play a role have received scant attention. It is the intersection of these two fields that forms the topic of this treatise.

Fractal-based point processes exhibit both the scaling properties of fractals and the discrete character of random point processes. These constructs are useful for representing a wide variety of diverse phenomena in the physical and biological sciences, from information-packet arrivals on a computer network to action-potential occurrences in a neural preparation.

Scope

The presentation begins with several concrete examples of fractals and point processes, without devoting undue attention to mathematical detail (*Chapter 1*). A brief introduction to fractals and chaos follows (*Chapter 2*). We then define point processes and consider a collection of measures useful in characterizing them (*Chapter 3*). This is followed by a number of salient examples of point processes (*Chapter 4*). With the concepts of fractals and point processes in hand, we proceed to integrate them (*Chapter 5*). Mathematical formulations for several important fractal-based point-process families are then set forth (*Chapters 6–10*). An exposition detailing how various operations modify such processes follows (*Chapter 11*). We then proceed to examine analysis and estimation techniques suitable for these processes (*Chapter 12*). Finally, we examine computer network traffic (*Chapter 13*), an important application used to illustrate the various approaches and models set forth in earlier chapters.

To facilitate the smooth flow of material, lengthy *Derivations* are relegated to *Appendix A*. *Problem Solutions* appear in *Appendix B*. For convenience, *Appendix C* contains a *List of Symbols*. A comprehensive *Bibliography* is provided.

Approach

We have been inspired by Feller's venerable and enduring *Introduction to Probability Theory and Its Applications* (1968; 1971) and Cox and Isham's concise but superb *Point Processes* (1980).

We provide an integrated exposition of fractal-based point processes, from definitions and measures to analysis and estimation. The material is set forth in a self-contained manner. We approach the topic from a practical and informal perspective — and with a distinct engineering bent. Chapters 3, 4, and 11 can serve as a comprehensive stand-alone introduction to point processes.

A number of important applications are examined in detail with the help of a canonical set of point processes drawn from biological signals and computer network traffic. This set includes action-potential sequences recorded from the retina, lateral geniculate nucleus, striate cortex, descending contralateral movement detector, and cochlea; as well as vesicular exocytosis and human-heartbeat sequences. We revisit these data sets throughout our presentation.

Other applications are drawn from a diverse collection of topics, including $1/f$ noise events in electronic devices and systems, trapping in amorphous semiconductors, semiconductor high-energy particle detectors, diffusion processes, error clustering in telephone networks, digital generation of $1/f^{\alpha}$ noise, photon statistics of Čerenkov radiation, power-law mass distributions, molecular evolution, and the statistics of earthquake occurrences.

Audience

Our exposition is addressed principally to students and researchers in the mathematical, physical, biological, psychological, social, and medical sciences who seek to understand, explain, and make use of the ever-growing roster of phenomena that are found to exhibit fractal and point-process characteristics. The reader is assumed to have a strong mathematical background and a solid grasp of probability theory. While not required, a rudimentary knowledge of fractals and a familiarity with point processes will prove useful.

This book will likely find use as a text for graduate-level courses in fields as diverse as statistics, electrical engineering, neuroscience, computer science, physics, and psychology. An extensive set of solved problems accompanies each chapter.

Website and Supplementary Material

Supplementary materials related to the practical aspects of data analysis and simulation are linked from the book's website. Errata are posted and readers are encouraged to contribute to the compilation. Kindly visit http://www.wiley.com/statistics/ and scroll down to the icon labeled "Download Software and Supplements for Wiley Math & Statistics Titles." Then find the entry "Lowen and Teich." Alternatively, you may directly access the authors' websites at http://cordelia.mclean.org/∼lowen/ and http://people.bu.edu/teich/.

Photo Credits

We express our appreciation to the many organizations that have provided assistance in connection with our efforts to assemble the photographs used at the beginnings of each chapter: Penck (courtesy of Bildarchiv der Österreichischen Nationalbibliothek, Vienna); Richardson (courtesy of Olaf K. F. Richardson); Cantor and Poincaré (courtesy of the Aldebaran Group for Astrophysics, Prague); Poisson, Yule, Pareto, Hurst, and Erlang [from Heyde & Seneta (2001), courtesy of Chris Heyde, Eugene Seneta, and Springer-Verlag]; Lapicque (courtesy of the National Library of Medicine); Cox (courtesy of Sir David R. Cox); Fourier (courtesy of John Wiley & Sons); Haar (courtesy of Akadémiai Kiadó, Budapest); Kolmogorov (courtesy of A. N. Shiryaev); Van Ness (courtesy of John W. Van Ness); Mandelbrot (courtesy of Benoit B. Mandelbrot); Gauss (S. Bendixen portrait, 1828); Lévy and Feller [from Reid (1982), courtesy of Ingram Olkin, Constance Reid, and Springer-Verlag]; Schottky (from the Schottky family album); Rice (courtesy of the IEEE History Center, Rutgers University); Neyman [from Reid (1982), courtesy of Constance Reid and Springer-Verlag]; Bartlett (courtesy of Walter Bird and Godfrey Argent); Bernoulli [frontispiece from Fleckenstein (1969), courtesy of Birkhäuser-Verlag]; Allan (courtesy of David W. Allan); Palm (courtesy of Jan Karlqvist, from the Olle Karlqvist family album). The photographs of Lowen and Teich were provided courtesy of Jeff Thiebauth and Boston University, respectively.

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STEVEN BRADLEY LOWEN

MALVIN CARL TEICH

Boston, Massachusetts